Maschinentafel hat nur endlich viele Zeilen. Es muss also eine Zeile geben, die als erste bei dem betrachteten Prozess zum zweiten Male massgebend ist. Da in den Zeilen das Verhalten vorgeschrieben ist, muss der Berechungsprozess von hier ab periodisch weiterlaufen."

On page 234:

"A heisst ein Teilwort von V, wenn es Worte C, D gibt, mit $B \equiv CAD$. Ist A ein Teilwort von B, so sind C, D hierbei im allgemeinen nicht eindeutig bestimmt (z. B. für $A \equiv \mid \mid$, $B \equiv \mid \mid \mid$ hat man $B \equiv \Box A \mid \equiv \mid A \Box$). Es gibt aber eindeutig eine Darstellung $B \equiv CAD$ mit kürzestem C. Diese soll die ausgezeichnete Zerlegung von B in bezug auf A heissen."

Here, the symbol "$\Box$" represents the null word.

The author uses a tightly packed notation which this reviewer finds pleasing. In particular, the representation of a compound Turing machine constructed from atomic ones is by means of a diagram about which the author remarks, ". . . erinnert an die 'Flussdiagramme' (oder 'Blockdiagramme'), die bei der Programmierung von elektronischen Rechenmaschinen verwendet werden." The diagrams are also reminiscent of the "state diagrams" of finite automata theory. An unhurried explanation is given of the conversion from a diagram to the Turing machine which it represents. By introducing easily remembered abbreviations, the author presents diagrams that are compact and perspicuous.

Chapter I is for the most part informal and includes historical remarks. Chapter II makes precise the notions of: Turing machine, Turing-decidability, -computability, -enumerability. Chapters III, IV, and V are devoted to proving the equivalence of Turing computability, μ-recursiveness (Kleene) and recursiveness (Herbrand-Gödel). Chapter VI covers unsolvability of the word problem for semi-Thue and Thue systems, undecidability of first order predicate logic, incompleteness of second order predicate logic (the proof is based on the former result), incompleteness of arithmetic. The final chapter (VII) includes brief treatments, in some cases the concept only, of the topics: Kleene hierarchy, universal Turing machines, λ-K definability, Fitch's minimal basic logic, Post canonical systems, Markov algorithms, and recursive analysis.

The remark on p. 233 concerning Post canonical systems (first two paragraphs) seems to the reviewer misleading. To "simulate" a logic by a Post canonical system often requires augmenting the alphabet of the given logic. One might, for example, introduce "W", "$\vdash$" as additional symbols to indicate "the following word is well formed" and "the following word is a theorem," respectively. See, for example, P. C. Rosenbloom, "Elements of Mathematical Logic," pp. 157–160.

As indicated by the topics mentioned above, the coverage is considerably less complete and the point of view more elementary than either Davis' "Computability and Unsolvability," or Kleene's "Introduction to Metamathematics."

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Systems: Research and Design. Edited by Donald P. Eckman. Wiley, New York, 1961. 310 pp. $8.50.

For the most part, the contributions of the fourteen joint authors of this book are almost-nonmathematical discussions of the general nature of systems and its various ideas and principles. Many authors are from industry and one is now a prominent member of the United States Department of Defense. The philosophical implications of systems research and design are discussed with gusto and from a variety of viewpoints. In the Editor's words, "Systems Research and Systems Design are composed of something old, something new and something borrowed." It is therefore surely not accidental that the binding of the book is blue.

Book Review Editor

Mathematical Methods in the Theory of Queueing. By A. Y. KHINTCHINE. Hafner, New York, 1960. 120 pp. $5.50.

This little volume by the eminent late Russian mathematician provides an excellent introduction to the theory of queues. The author's emphasis is not on a detailed treatment of numerous special problems, but rather on the "main ideas, methods, and different ways of thought which govern the application of the theory of probability to questions of mass service." The exposition is admirably clear and the book should be accessible to "anyone who has mastered the main concepts of the theory of probability and followed some course however short in mathematical analysis." Nevertheless, it must be noted that the treatment throughout is decidedly mathematical. Basic concepts and theorems are given a very detailed and rigorous analysis, and the reader who is interested in queues primarily for their application, may at times become impatient.

Part One, which constitutes more than a third of the book is devoted to a discussion of the input process (apart from all considerations of service). The author begins by introducing the familiar Poisson process (it is referred to here as a "simple stream") as one satisfying the conditions of (1) stationarity: for any $t > 0$ and integer $k \geqq 0$ the probability $V_k(t)$ that during the time $(a, a + t)$ there occur k events is the same for all $a \geqq 0$; (2) absence of after-effects: $V_k(t)$, above, is independent of the sequence of events up to the moment a; (3) orderliness:

$$\lim_{t \to 0} \frac{1}{t} \sum_{k=2}^{\infty} V_k(t) = 0;$$

i.e., roughly speaking, two or more events occur at the same moment of time. The first four chapters treat the simple stream and the more general streams which result when one or more of the above conditions are dropped. The first part concludes with an important limit theorem giving conditions under which the superposition of n stationary and orderly streams results (as $n \to \infty$) in a simple stream.

Part Two discusses system with losses, i.e., systems where no waiting is permitted. Under the assumption of a Poisson input and exponentially distributed service time various probabilities are obtained both for the case of n servers and the simpler case of an infinite number of servers. Here, as elsewhere, the author gives considerable attention to the proof of fundamental results, such as the existence of a stationary state.

The final part treats systems where delay is allowed, first under the assumption of an exponentially distributed service time, and then for a constant service time. The book ends with a beautiful derivation of the classical Khintchine-Pollaczek formula giving the characteristic function of the time of waiting.